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$$5. 48\frac{17}{29} \times 49\frac{12}{29} = 2400\frac{697}{841}; \quad 1295\frac{57}{93} \times 1296\frac{36}{93} = 1677615\frac{7343}{8649}.$$

$$6. 7464 \times 7536 = 56248704; \quad 88044 \times 87956 = 7744000000 - 1936.$$

$$7. 2777\frac{7}{9} \times 4166\frac{2}{3} \times 666\frac{2}{3} \times 54 \times 24 \times 52 \times 7692307\frac{9}{13} \times 625 \times 125 \times 56 \times 32 \times 1428571428571428\frac{4}{7} \times 2083\frac{1}{3} \times 48 \times 833\frac{1}{3} \times 3125 \times 68543764287590 = \dots''$$

These and eleven other even longer computations are carried out mentally by Mr. Case.

The principle,  $a^2 = (a-b)(a+b) + b^2$ , may be used in the squaring of any number, though it is not so readily used if the numbers consist of more than two digits. Thus,

$$87^2 = (87-3)(87+3) + 3^2 = 84 \times 90 + 9,$$

$$92^2 = (92-2)(92+2) + 2^2 = 90 \times 94 + 4.$$

This is the principle used in several of Mr. Case's calculations. Thus,  $(5\frac{1}{2})^2 = (5\frac{1}{2} - \frac{1}{2})(5\frac{1}{2} + \frac{1}{2}) + (\frac{1}{2})^2 = 30\frac{1}{4}$ . ED. F.

330. Proposed by R. D. CARMICHAEL, Princeton, N. J.

An important function in the Theory of Numbers is one defined thus:  $f(x)=1$  when  $x>0$ ,  $f(x)=0$  when  $x=0$ ,  $f(x)=-1$  when  $x<0$ . Two analytic expressions for  $f(x)$  are the following:

$$f(x) = \lim_{n \rightarrow \infty} x^{1/(2n-1)}, \quad n=1, 2, \dots; \quad f(x) = \lim_{n \rightarrow \infty} \frac{(x+1)^n - (x+1)^{-n}}{(x+1)^n + (x+1)^{-n}}, \quad x > -1.$$

It is required to find other non-trigonometric analytic expressions for this function. (There are several representations of  $f(x)$  by means of trigonometric functions.)

Remark by the PROPOSER.

Professor F. H. Safford, of the University of Pennsylvania, has sent me the following expressions for the function defined in the problem:

$$\frac{2}{\pi} \int_0^\infty \frac{\sin xz}{z} dz, \quad \frac{2}{\pi} \int_0^\infty \frac{x dz}{x^2 + z^2}, \quad \text{Lim.}_{m \rightarrow +\infty} \frac{e^{xm} - e^{-xm}}{e^{xm} + e^{-xm}}.$$

333. Proposed by R. D. CARMICHAEL, Princeton University.

Sum the infinite series

$$\frac{1}{(m+1)^2} + \frac{(2m-1)}{(2m+1)^2} + \frac{(3m-1)^2}{(3m+1)^4} + \frac{(4m-1)^3}{(4m+1)^5} + \frac{(5m-1)^4}{(5m+1)^6} + \dots$$

[No solution of this problem has been received.]

334. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

$$\text{Sum the series, } 2^n - n \cdot 2^{n-2} + \frac{n(n-3)}{2!} 2^{n-4} - \frac{n(n-4)(n-5)}{3!} 2^{n-6} \\ + \frac{n(n-5)(n-6)(n-7)}{4!} 2^{n-8} - \frac{n(n-6)(n-7)(n-8)(n-9)}{5!} 2^{n-10} + \dots$$

**Solution by the PROPOSER.**

We have,  $(1-px)(1-qx) \equiv 1-x(p+q-pqx)$ . Taking logarithms,

$$\log(1-px) + \log(1-qx) = \log[1-x(p+q-pqx)].$$

$$\begin{aligned} \text{Hence, } (p+q)x + \frac{(p^2+q^2)x^2}{2} + \frac{(p^3+q^3)x^3}{3} + \dots + \frac{(p^n+q^n)x^n}{n} \\ = x(p+q-pqx) + \frac{x^2(p+q-pqx)^2}{2} + \dots + \frac{x^n(p+q-pqx)^n}{n}. \end{aligned}$$

The coefficients of  $x^n$  are equal for values of  $x$ , which makes the series convergent.

$$\begin{aligned} \therefore \frac{p^n+q^n}{n} &= \frac{(p+q)^n}{n} - \frac{(n-1)pq(p+q)^{n-2}}{n-1} \\ &+ \frac{(n-2)(n-3)p^2q^2(p+q)^{n-4}}{2!(n-2)} - \frac{(n-3)(n-4)(n-5)p^3q^3(p+q)^{n-6}}{3!(n-3)} + \dots \end{aligned}$$

$$\text{Hence, } p^n+q^n = (p+q)^n - npq(p+q)^{n-2}$$

$$+ \frac{n(n-3)}{2!} p^2q^2(p+q)^{n-4} - \frac{n(n-4)(n-5)}{3!} p^3q^3(p+q)^{n-6} + \dots$$

$$\text{Let } p=q. \quad \text{Then } 2 = 2^n - n2^{n-2} + \frac{n(n-3)}{2!} 2^{n-4} - \frac{n(n-4)(n-5)}{3!} 2^{n-6}$$

$$+ \frac{n(n-5)(n-6)(n-7)}{4!} 2^{n-8} - \dots$$

Solved similarly by V. M. Spunar, who starts with the identity,  $2\log(1-x) = \log(1-2x+x^2)$ .